In this latter
picture a par
corresponds to
ray ending u
exactly n la
position remove
from its orig
position.

① pick a direction vector
② calculate reflection off 3 sets of orthogonal plan
③ calculate travel distance between successf
hits on a single set of planes
④ assign a loss/cm number

this looks amenable to an analytic
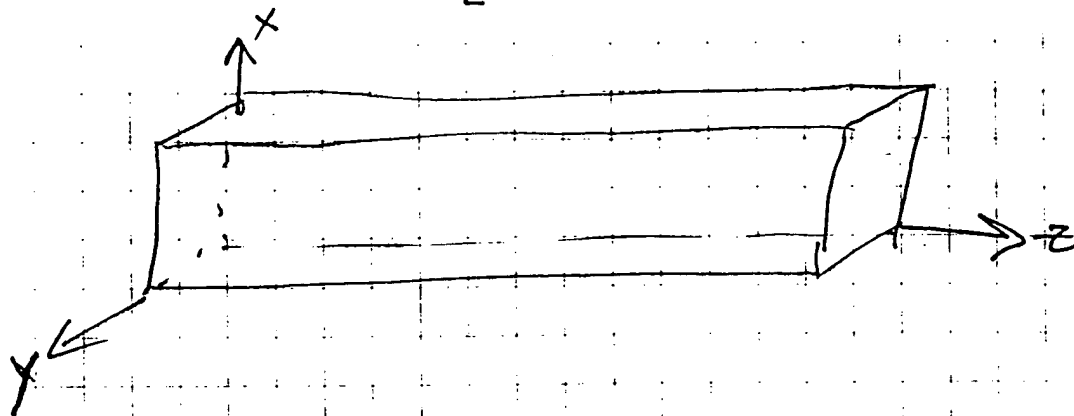treatment

use direction cosines to parameterize ray direction
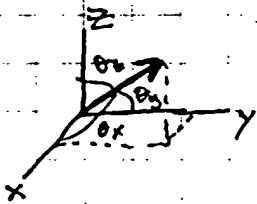
$$(\cos\theta_x, \cos\theta_y, \cos\theta_z) = \frac{(RND1, RND2, RND3)}{\sqrt{RND1^2 + RND2^2 + RND3^2}}$$

Let Δx, Δy, and Δz denote slab dimensions or
plane spacing.

6-Sep-9

From this point of view = it doesn't matter
what position a ray is launched from, onl
its direction, because launch position has
no impact on spacing between plan
strikes

Treat cants on surface perturbatively.

2 questions:
① How big do cants have to be to eliminate all parasitics?

② For a rectangular slab, how close to slab index does cladding index have to be to eliminate all parasitics. This question will be easiest to answer for a zero loss parasitic.

set z face incident angle equal to $\theta_{crit z} = \sin^{-1}(\frac{1}{n_s})$ and the x face hit and y face hit also $= \theta_{crit z} = \sin^{-1}(\frac{n_c}{n_s})$

make this argument more rigorous!

Now work with direction cosines

$$\theta_{c1} = \sin^{-1}\left(\frac{1}{n_s}\right)$$

$$\theta_{c2} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$$

$$\cos \theta_{c1} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

$$\cos \theta_{c2} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos^2 \theta_{c1} + 2\cos^2 \theta_{c2} = 1$$

$$\frac{n_s^2 - 1}{n_s^2} + \frac{2(n_s^2 - n_c^2)}{n_s^2} = 1$$

$$n_s^2 - 1 + 2n_s^2 - 2n_c^2 = n_s^2$$

$$2(n_s^2 - n_c^2) = 1$$

$$n_s^2 - n_c^2 = \frac{1}{2}$$

when can this no longer be solved

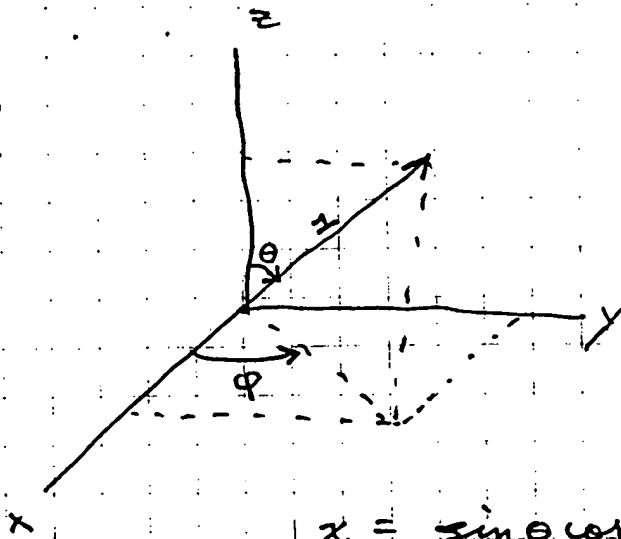$$n_c = \sqrt{n_s^2 - \frac{1}{2}}$$

$$n_c = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

for $n_c > 1.677$ no zero loss parasitics exist!

↳ This agrees with
code: slab ASE 01.XCL
prediction

Question 2 will be easiest to answer numerically: finding the angular width over which a parasitic exists for given gain and cladding indices.

$$x = \sin\theta\cos\varphi = \cos\theta_x$$
$$y = \sin\theta\sin\varphi = \cos\theta_y$$
$$z = \cos\theta = \cos\theta_z$$

$\theta_x < \theta_{x\text{-crit}}$
$\theta_y < \theta_{y\text{-crit}}$
$\theta_z < \theta_{z\text{-crit}}$ $\Big\rangle$ to avoid 0-loss parasitics

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$\cos\theta_x > \cos\theta_{x\text{-crit}}$
$\cos\theta_y > \cos\theta_{y\text{-crit}}$
$\cos\theta_z > \cos\theta_{z\text{-crit}}$ $\Big\rangle$ to avoid 0-loss parasitics

$\sin\theta_{x\text{-crit}} = \dfrac{n_c}{n_s}$
$\sin\theta_{y\text{-crit}} = \dfrac{n_c}{n_s}$
$\sin\theta_{z\text{-crit}} = \dfrac{1}{n_s}$

$$\cos\theta_x > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos\theta_y > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos\theta_z > \frac{\sqrt{n_s^2 - 1}}{n_s}$$ $\Big\rangle$ to avoid 0-loss parasi

$$1 = \cos\theta_x^2 + \cos\theta_y^2 + \cos\theta_z^2 \text{ and this must be greater than}$$

$$1 > \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$ to avoid 0-loss parasiti

$$1 > \frac{3n_s^2 - 2n_c^2 - 1}{n_s^2}$$

n_s is slab index
n_c is coating index
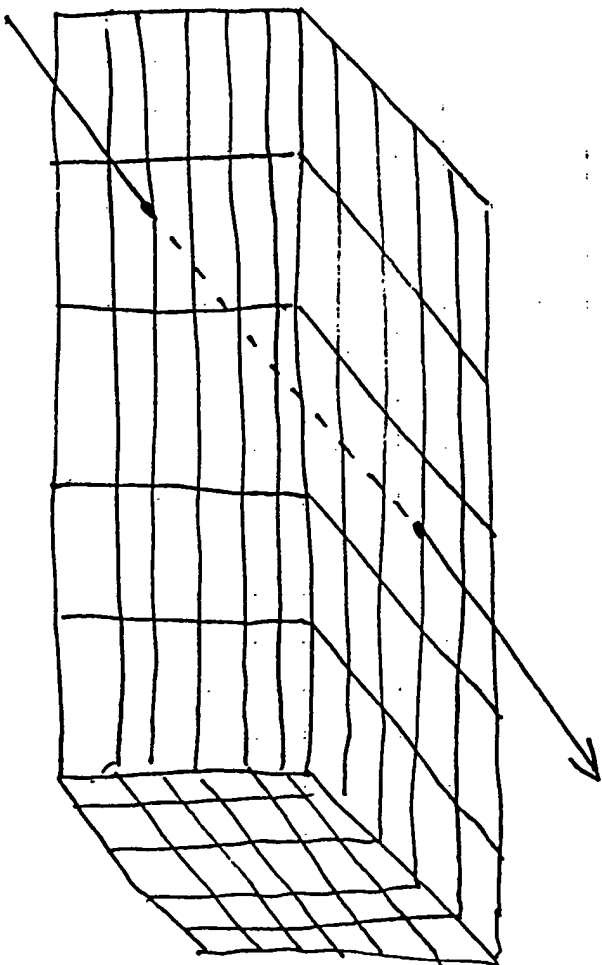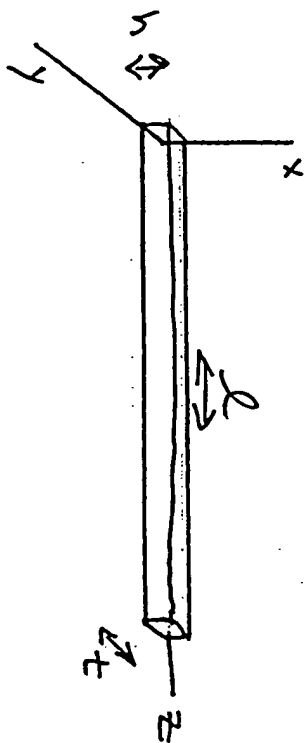
$$n_s^2 > 3n_s^2 - 2n_c^2 - 1$$

$$1 > 2(n_s^2 - n_c^2)$$

$$\frac{1}{2} > n_s^2 - n_c^2$$

$$n_c^2 > n_s^2 - \frac{1}{2}$$

$$n_c > \sqrt{n_s^2 - \frac{1}{2}} = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

using a method of images construction

fill space
using slab
and
real-images

- define arbitrary ray direction using direction cosines $(\cos\theta_x, \cos\theta_y, \cos\theta_z)$
- gain of ray in given in nepers/cm by

$$g = \frac{\ln(Ref_x)}{(1/\cos\theta_x)} + \frac{\ln(Ref_y)}{(t/\cos\theta_y)} + \frac{\ln(Ref_z)}{(\ell/\cos\theta_z)}$$

where: Ref_i is the reflection coefficient for i-oriented planes

$\frac{N_g}{N_g}$ is the specific gain $\frac{Corr}{Corr}$

zero - loss parasitics correspond to those may directions that are confined by TIR at all three sets of planes

$$\cos\theta_x < \cos\theta_{x\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\left.\cos\theta_y < \cos\theta_{y\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}\right\} \text{TIR condition}$$

$$\cos\theta_z < \cos\theta_{z\text{-crit}} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

where:
n_s = slab index
n_c = coating index

- Since $1 = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z$, zero-loss parasitics exist when

$$1 < \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

or

$$\boxed{n_c < \sqrt{n_s^2 - \frac{1}{2}}}$$

Zero-loss parasitics can be completely suppressed by choosing a cladding with refractive index large enough

$$\boxed{n_c > \sqrt{n_s^2 - 1}}$$